Through a Lens Darkly: Evidence for Dusty Gravitational Lenses

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ABSTRACT
Foreground galaxies that amplify the light from background quasars may also dim that light if the galaxies contain enough dust. Extinction by dust in lenses could hide the large number of lensed systems predicted for a flat universe with a large value of the cosmological constant Λ. We look for one signature of dust, namely reddening, by examining optical-infrared colors of gravitationally lensed images of quasars. We find that the lensed systems identified in radio and infrared searches have redder optical-IR colors than optically selected ones. This could be due to a bias against selecting reddened (hence extincted) quasars in the optical surveys, or due to the differences in the intrinsic colors of optical and radio quasars. Comparison of the radio-selected lensed and unlensed quasars shows that the lensed ones have redder colors. We therefore conclude that at least part of the color difference between the two lens samples is due to dust.

From the color difference between lensed and unlensed radio quasars (and assuming Galactic extinction law) we can reconcile a large cosmological constant ($\Lambda = 0.9$) with the number of lensed systems observed in flux limited optical surveys. These results substantially weaken the strongest constraint on cosmological scenarios that invoke a non-zero cosmological constant to explain age discrepancy problems, satisfy predictions of inflationary models of the early universe and play a role in large scale structure formation models. They also raise the prospect of using gravitational lenses to study the interstellar medium in high redshift galaxies.

Key words: Cosmology: gravitational lensing, Cosmology: observations, Cosmology: large-scale structure of the Universe, galaxies: ISM, ISM: dust.

1 INTRODUCTION

The statistics of gravitational lensing provide some of the strongest upper limits on the cosmological constant Λ (Turner 1990, Fukugita, Futamase, & Kasai 1990), one of the three parameters (Ω, Λ, H_0) that determine the geometry and the age of the universe (Carroll, Press & Turner 1992). Models with large values of Λ predict many more lensed objects than are presently observed in optical surveys (Maoz & Rix 1993, Kochanek 1993). The constraints from radio surveys and radio lenses are not so stringent because the redshifts and the luminosity function of radio sources are not well determined (Kochanek 1996).

The idea that the dust in lensing galaxies can be an important factor is fairly recent. Fukugita & Peebles 1993 calculated the effect of dust on lensing statistics by making all ellipticals dusty beyond a redshift of 0.5. Kochanek (1996) showed that one would need an average reddening of

2 magnitudes to reconcile the lensing statistics with a high Λ universe. Lawrence et al. (1995) and Larkin et al. (1994) show examples of heavily reddened lensed quasars found with the MIT-Green Bank survey and Nadeau et al. 1991 demonstrated differential reddening between the images of Q2237+0305, which is lensed by a spiral. The idea of dust in lensing galaxies may have been slow in coming because most of the lensing at arcseconds scales is due to elliptical and S0 galaxies (Turner Ostriker & Gott 1984 (TOG84)) believed to have little or no interstellar matter at the present epoch. There is however evidence for some dust and gas in present day early type galaxies. More than 50% of elliptical and S0 galaxies are detected in dust emission with IRAS (Knapp et al. 1989) and $\simeq$ 40% of elliptical galaxies from the Revised Shapely-Ames catalog have dust lanes or patches at optical depths $0.1 < \tau_V < 0.7$ (Goudfrooij & de Jong 1995). Dust lanes were seen in the cores of $78 \, {}^{+}_{-} \, 16\%$ of elliptical galaxies observed with HST (van Dokkum & Franx 1995).

Besides misleading us about the total number of gravitational lenses, dust in lenses may bias our conclusions regarding the relative number of lenses as a function of image separation, another diagnostic of cosmology and structure formation models (TOG84, Wambsganss et al. 1995). Also, higher redshift lensing galaxies will have more extinction for the same amount of dust because they see the quasar light at shorter wavelengths. This may bias the tests for Λ based on the redshifts of the lensing galaxies (Kochanek 1992).

In section 2 we report a purely empirical test to determine whether there is dust in lenses by comparing the optical-IR colors of radio-selected lens systems with those of optically selected systems. Dust transmits redder wavelengths preferentially, so objects seen through large amounts of dust are reddened and dimmed in optical wavelengths. The faint, reddened images may easily be missed by optical surveys. At radio wavelengths, dust is transparent, and we expect radio surveys to contain lens systems with all degrees of reddening, without bias. The radio selected sample should therefore be redder if there is a significant amount of dust in the lensing galaxies.

In presenting evidence for reddening of the lensed image due to the intervening lensing galaxy, we need to address two issues: (1) Are the lensed radio-selected quasars redder than the optically selected lensed quasars, since we expect not to be able to observe many reddened optical lenses? (2) Are these lenses redder than the unlensed population they are derived from? These two questions are related; the optical sample from which the lenses are identified may be intrinsically less red than the radio sample, and that could well explain the color difference between the two lensed samples. In section 3 we address the second question by comparing the optical-IR colors of lensed and unlensed radio and optical quasars.

In section 4 we discuss other effects besides dust reddening which can result in red optical-IR colors; some other evidence for dust and interstellar medium in lensing galaxies; implications for using lensing optical depth as a test for Λ; and further tests to verify the presence of dust in lensing galaxies.

2   OPTICAL-INFRARED COLORS OF LENSED QUASARS

We compare the optical-infrared colors of the radio sample with the optical sample. The infrared and optical photometry of an almost complete sample of known and suspected cases of multiply imaged objects (Keeton & Kochanek 1996, Surdej 1996) is compiled partly from an infrared survey of gravitational lenses carried out at Apache Point Observatory (Rhoads et al. 1997) and partly from the literature (Soifer et al., Lawrence et al. 1995, Annis & Luppino 1993, Larkin et al. 1994, Nadeau et al. 1991, Lawrence et al. 1992, 1995, Barvainis et al. 1995, Rowan-Robinson et al 1993, McLeod, Rieke & Weedman 1994, Tinney 1995).

Figure 1 shows the distribution of the optical-IR2 colors and the IR1-IR2 colors of the two samples. The optical band is mostly R and the two infrared bands are J(IR1) and H or K (IR2). The wavelengths observed are slightly inhomogeneous but that is negligible compared to the wide range of emission wavelengths due to the broad redshift distribution
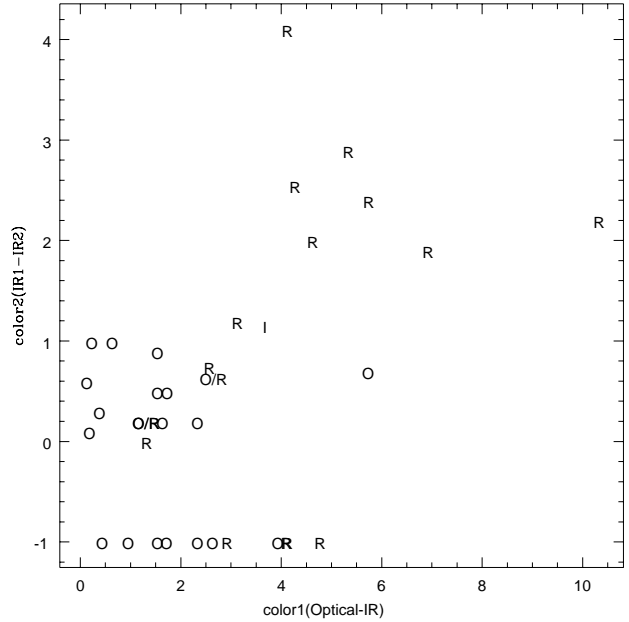

Figure 1. The optical-infrared and infrared-infrared colors of known and suspected multiply imaged gravitationally lensed objects. The optical band is mostly R and sometimes r or i. The near infrared bands are J for IR1 and H or K for IR2. The radio selected objects are represented by R and optically selected objects are denoted by O. I denotes the sole infrared selected object IRAS F10214+4724. We see that compared to the radio selected sources optically selected lenses are systematically bluer in both colors. In fact the color distribution of optical and radio samples are almost disjoint. The trends are consistent in the two colors shown, i.e. objects red in Optical-IR2 color are red in IR1-IR2 color as well. Where we have a measurement of only one infrared band we denote the IR1-IR2 color as -1.

of the sources. We also include in the sample a single lens, FS10214+4724, found in the IRAS survey (and formerly the most luminous object in the universe) and treat it as a radio selected source since it was not subject to the biases that the optical surveys may have been.

It is clear from Figure 1 that the radio selected lensed images are redder than the optically selected ones. The optically selected and radio-selected quasars have almost disjoint distributions in optical-IR colors. The Wilcoxon test (Lupton 1993) for the means of the two distributions shows that the mean optical-IR color of the radio selected sample is different from the color of the optical sample at 99.99% confidence level. The IR1-IR2 colors are likewise very different in the different samples.

We have used each resolved image or unresolved group of images as an independent point, which is justifiable if the different color is due to reddening and each image goes through a different path in the lensing galaxy. If the color difference between the radio and optical samples is due to the intrinsic color difference in the background sources, then each source, and not each image, is an independent data point. In that case the mean optical-IR colors of the two samples differ at the 99.7% confidence level.

Since the different lensed objects lie at different redshifts and are measured in a slightly inhomogeneous set of
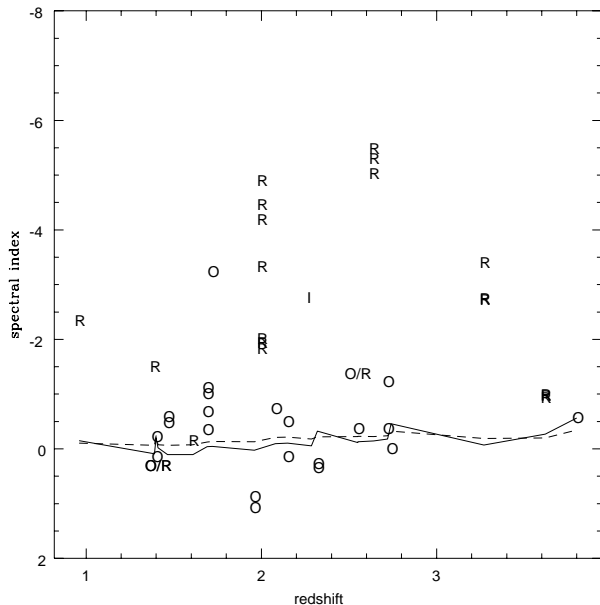
Figure 2. The spectral index of the lensed quasars is plotted against their redshift. The radio selected objects are represented by R and optically selected objects are denoted by O. I denotes the sole infrared selected object IRAS F10214+4724. The solid line shows the spectral index for the lensed object if it had the mean SED (Elvis et al. 1994) of an unlensed radio-loud quasar. The dotted line shows the same quantity if the source was a radio-quiet quasar. The optically selected lensed quasars have spectral indices clustered around the mean index of unlensed quasars, and the radio selected sample shows steeper spectra. For some sources the redshifts are not known; they are plotted here at a redshift of 2.

bands, the best way to compare their colors is by means of the spectral index α, where $f_\nu \propto \nu^\alpha$ and f_ν is the flux density at frequency ν. (This is a useful quantity because quasar spectra are power laws to a good approximation at these wavelengths). Since most of the sources are at redshifts between 1 and 4, we are actually measuring ultraviolet-optical colors in the source rest frame. Figure 2 shows the spectral indices for the radio and the optical samples. We also calculate the spectral indices that would be expected for each lensed object if it had the mean spectral energy distribution of an unlensed radio-quiet or radio-loud quasar (Elvis et al. 1994).

We see that the radio sample has steeper spectral indices than the optical sample and that the redshift distribution of the two samples is very similar (Wilcoxon test shows that the mean redshifts differ only at the 6% confidence level). So it is unlikely that any special spectral features contribute to the color difference of the radio and the optical samples. The spectral indices derived from the optical lensed sample in general match well with those derived from the mean Spectral Energy Distribution (SED) from both the radio loud and the radio quiet samples, whereas the spectral indices of the radio lensed sample indicate redder colors.

3 COMPARISON WITH UNLENSED QUASARS

The lens systems come from many surveys and other observations, notably optical surveys PG (Green, Schmidt & Liebert 1986), LBQS (Hewett et al 1995), and radio surveys MG (Burke et al. 1992), JVAS (Patnaik 1994, King et al. 1996), CLASS (Myers 1996). Because lensed sources are magnified we cannot present an exhaustive survey of the optical-IR colors or optical spectral indices of these surveys down to the intrinsic faintness of the lensed objects. We can however compare the spectral indices (and hence the colors) of the lensed samples with some unlensed objects in the samples (Sanders et al. 1989, Francis, 1995). None of the radio surveys have systematic measurements in optical and IR. The JVAS survey consists of flat spectrum radio sources and the MG survey does not discriminate on the basis of spectral index. We use two samples of unlensed quasars with flat and steep radio spectra (Webster et al. 1996, Dunlop et al 1989) . Comparing the spectral indices derived from these two samples separately by Wilcoxon test, we conclude that the radio selected lensed quasars have steeper spectral indices than the radio selected quasars at the 99.99% confidence level. The optically selected lensed quasars show no significant difference in their spectral indices as compared to the unlensed sample.

Figure 3 shows the distribution of the spectral indices for the 6 samples: The Palomar Green (PG) survey, Large Bright Quasar Survey (LBQS), optically selected lensed quasars, the Parkes radio quasars (Webster et al. 1995, Dunlop et al. 1995) and the radio selected lensed objects. The spectral indices were computed (mostly) from $R(0.7\mu\,m)$ and $K(2.2\mu\,m)$ broadband photometry. The optically selected lenses and PG quasars are not very different in their spectral indices. The radio selected lensed sample differs significantly from the unlensed Parkes survey sample, with the spectral index being steeper (implying redder colors) for the lensed quasars. The (unlensed) radio quasars are redder than the optical PG and LBQS quasars. Webster et al. (1995) interpret the red color of some of the Parkes quasars as evidence of dust reddening between the quasar and the observer. This is consistent with reddening being more frequent for lensed objects, since their light has *a posteriori* a much higher chance of encountering a fully formed galaxy with dust in it.

4 DISCUSSION

Lensed images might also appear red if their light is contaminated by light from the lensing galaxy, or if the background source is a high redshift radio galaxy. These do not appear to be large effects in our sample: out of 27 lens systems, six are unresolved; in two the background source is a radio galaxy; and in two the background source is unidentified. These cases are not the reddest images in the sample, and removing them from the sample does not change the above results. Blazars are the reddest flat radio spectrum sources, because doppler boosted synchrotron radiation contributes to the red part of their spectra. The steepest optical-IR spectral index for such objects (Impey, Lawrence & Tapia 1991) is $\alpha = -2.6$, a value which is merely average for lensed radio quasars (Figure 3). Among lensed radio sample, only
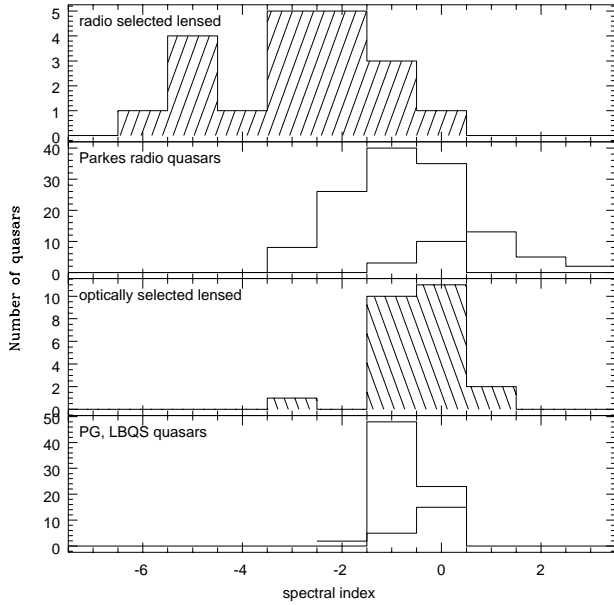
Figure 3. A histogram of the distribution of spectral indices in various samples of quasars. Going from the bottom panel to the top we plot optical unlensed quasars, optical lensed quasars, radio unlensed quasars (with both flat and steep radio spectra), and radio selected lensed systems. The spectral indices are calculated from broadband measurements in R and K bands mostly. While there is not much difference in the spectral indices of lensed and unlensed optical quasars, the radio samples of lensed and unlensed quasars differ significantly.

B 0218+357 is known to be a blazar, and there is other evidence for extinction in that system (see below). Starlight from the quasar host galaxy is not important at the rest-wavelengths (0.6 μ m corresponding to measured K-band) of these high reshift lensed quasars, because the total light from the host galaxy is $\sim$ 2–3 magnitudes fainter than the quasar (cf. Bahcall, Kirhakos & Schneider 1996; Mcleod & Rieke 1995 and references therein) and not all of the host galaxy is in the strongly lensed portion of the source plane. Moreover, the host galaxy light contributes similarly in radio-loud and radio-quiet quasars (Dunlop et al. 1993).

4.1   Estimating the column density of dust

We could estimate the dust in the lensing galaxy if we knew the intrinsic spectrum of the source. Lack of knowledge of the reshifts of sources and many lensing galaxies adds to the uncertainty. Because the sources are at high redshifts, measured optical-IR colors correspond to ultraviolet colors in the rest frame of the sources and optical colors at the redshifts of the lensing galaxy; hence huge amounts of dust are not needed to produce the observed reddening. If we assume that the intrinsic spectrum of the quasars is given by the mean SED (Elvis et al. 1994) and the dust is similar to the Galactic dust (Draine & Lee 1984), we estimate $A_v = 0.3 - 4$ magnitudes for those lens systems which have the redshifts of both the lens and the source measured.

B 0218+357 illustrates this point nicely. The ratio of the brightness of images A/B is much smaller in optical than in radio (Grundahl & Hjorth 1995). Interferometric observations with the VLA (Menten & Reid, 1996) show absorption by H_2CO which occurs in dense molecular gas *only* against component A and derive a column density of N(H)= $10^{21} - 10^{22} cm^{-2}$, assuming Galactic abundance of H_2CO. From CO observations Wiklind & Combes (1995) derive a lower limit to the column density of $N(H_2) > 1.5 \times 10^{21} cm^{-2}$. This, along with HI column density N(H)= $10^{18} T_{spin} cm^{-2}$ (Carilli, Rupen & Yanny 1993), gives $A_V = 1.9 - 12$ magnitudes for dust and gas similar to Galactic. The discrepancy between the optical and radio ratios of A and B brightness imply a differential extinction of 3.7 magnitudes between A and B. From the reddening seen in Q0218+357 and assuming the mean SED (Elvis et al. 1994) we derive a luminosity weighted average extinction in the lens to be 4 magnitudes.

Lawrence et al. (1995) make a detailed case for *MG 0414+0534* being obscured by dust, and it is indeed the reddest known radio-selected lens system followed by *1938+666* (Rhoads, Malhotra & Turner 1997). Recent observations of time variation in colors support the dust-in-the-lens hypothesis for *MG 0414+0534* (Vanderriest et al. 1996). Nadeau et al. (1991) present evidence for differential reddening of different images of the quasar in *Q 2237+0305* and derive a reddening law similar to the Galactic reddening law. Wiklind and Combes (1996) find high (N(H)= 3×10^{22}) column density of molecular gas in absorption against one of the images of *PKS 1830-210* at redshift of 0.89 and Lovell et al. (1996) find HI in absorption at z=0.19. (The optical counterpart to this lens is not known and this radio selected lens is not included in our analysis.)

Besides these lenses, there are instances of extinction of background sources by elliptical galaxies (Stocke et al. 1984, Stickel et al. 1996, McHardy et al. 1994). To our knowledge there is no systematic study searching for such a phenomenon at high redshifts, and these cases have been identified because of some oddity in their behavior at other wavelengths.

4.2   How dust affects lensing constraints on Λ

To see how dust can affect the constraints on Λ derived from statistics of optically selected lens systems, we need to quantify the differences in color (or spectral indices) between lensed and unlensed radio selected samples. This could be done by determining the transfer function that maps the distribution of spectral indices of the unlensed sample to that of the lensed sample (top two panels of figure 3). Ideally this should be done for a large number of lensed quasars and their parent samples. The transfer function could then be applied to the optical quasar sample to see how many reddened and extincted optical quasars exist and are missing from the surveys.

Such a detailed treatment is not justified at present for three reasons. First, the transfer function would be noisy because the number of lensed images is small. Second, we don't really have the color distribution of the parent sample of unlensed radio quasars for all the radio lens surveys (MG, JVAS, CLASS). And third, if color is a function of magnitude, we would need to know both the color distribution of sources down to the intrinsic brightness of the lensed sources (which is fainter than the survey limits) and the magnification of each system.

Instead, we use a much simpler procedure of calculating the average shift in spectral indices between the radio lensed and unlensed samples. This procedure should give an idea of how much a population is affected by dust in lenses. The average shift of spectral index, $\delta\alpha = 1.9^+_-0.27$, translates into different amounts of dust extinction depending on the redshift of the lens. We estimate an average extinction of $A_v = 1.0^+_-0.55$ for the radio selected lensed quasars (where the uncertainty includes the spread in A_v due to different redshifts of the lensing galaxy).

Next we apply the extinction effects to the calculation of expected number of lensed quasars in the AAT survey in the optical (Boyle et al. 1988). Malhotra and Turner (1995) estimated that in the AAT survey there should be 3.5 lensed systems for a $(\Omega = 0.1, \Lambda = 0.9)$ cosmology and 0.5 systems for $(\Omega = 1, \Lambda = 0)$ cosmology. This calculation takes into account the different luminosity functions for unlensed quasars inferred in different cosmologies and uses reduced lensing cross-sections from Fukugita & Turner (1991). An average extinction of $A_v = 1^+_-0.5$ implies an extinction of 2^+_-1 magnitudes of in the U-band, the wavelength at which a $z = 0.5$ lensing galaxy sees the B-band light seen by the observer. The expected number of lensed systems then drops to 0.25–1.5 systems for a $(\Omega = 0.1, \Lambda = 0.9)$ cosmology, which is consistent with the AAT survey not having found a lensed system.

The difference in the inferred reddening between radio-selected and optically selected lens systems may not be completely attributable to a bias against finding reddened lens systems in the optical. Radio searches are better able to detect small-separation lenses, and spiral galaxies contribute about equally to the lensing optical depth at angular separation of $< 0.5''$ (TOG84). So reddening may be more significant for small separation lenses. We do not, however, see a strong correlation of red images and image separation in our sample.

4.3 Confirmation of dust in the lensing galaxies

The confirmation of dust in lenses could come in various ways, all of which assume some similarity between dust and gas at high redshifts and in our Galaxy: (a) Detection of gas in the lensing galaxies suggests dust may be present, especially if CO is detected, since CO implies reasonable metallicities. (b) Detection of differential reddening between images of the same quasar allows the color difference to be tested for consistency with known reddening laws, independent of the (unknown) intrinsic quasar spectrum (cf. Nadeau et al. 1991). Because the magnification ratios and differential dust column densities are imperfectly known, good photometry of individual lens components in 3 bands is needed for this test. Near-IR bands are best for this purpose because the Galactic reddening law is the least variable at these wavelengths (Whittet 1988). If the relative colors of different images agree with the Galactic reddening law, one can estimate the differential extinction between the images. Correcting for this differential extinction will improve magnification ratio measurements for different images and hence improve the lens models. (c) Observation of known, strong spectral features of dust will perhaps be the most convincing evidence. The $2175\text{\AA}$ absorption bump (cf. Fitzpatrick & Massa 1986) is a strong feature in the UV and should be

detectable for small column densities of dust (large column densities of dust would rapidly produce too much extinction in the UV). The silicate absorption feature at 9.7μm, on the other hand, should be ideal for large column densities of dust (Roche & Aitken, 1984) because the continuum extinction at these wavelengths is small. The absorption features have the advantage that their detectability depends only on the brightness of the background source (which is amplified by lensing) and the column density of the dust (which is exactly what we would like to measure).

4.4 Conclusions

The dramatic systematic difference in the optical-IR colors between radio and optically selected gravitational lens systems and between radio lensed and unlensed quasars reported here, and the other evidence for dust in lensing galaxies discussed above, strongly suggest (but do not prove) that optical searches for lens systems are seriously incomplete due to extinction. If so, the limits on Λ deduced from lensing statistics from optical surveys (Maoz & Rix 1993, Kochanek 1993) are significantly too stringent, and the most serious objection to invoking non-zero Λ models to resolve a variety of cosmological puzzles (Carroll et al. 1993, Ostriker & Steinhardt 1994) can be relaxed.

If the optical searches are biased against images extincted by lenses, and background radio sources are fewer in number than optical sources, searches in the near infrared should be most successful in finding lens systems. Stickel et al. 1996 point out that of a group of 30 radio sources picked out for being unusual in their steep optical index by Rieke et al. (1979, 1982) and Smith & Spinrad (1980) two (B 0218+357 and MG 0414+0534) were later identified as being gravitationally lensed. Usual fraction of lenses in an optical or radio surveys is about 1 or 2 systems per 1000 quasars! The reddened systems can be used to study the interstellar medium and dust at high redshifts, as is being done now (Wiklind & Combes, 1995, 1996, Carilli et al. 1993, Menten & Reid, 1996, Nadeau et al. 1991). Gravitational lensing thus has the potential to contribute to our knowledge of interstellar matter and dust in the high redshift galaxies, as well as the gravitational potential of galaxies and clusters and the geometry of the universe.

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